Lecture 3

- Answer questions from yesterday

Three factors are important.

- The valence quarks move relativistically
- Virtual excitation of anti-quarks in low-lying P-states via one-gluon-exchange.
  In nuclear physics terminology— exchange current corrections.
- The pion cloud of the nucleon.

The cloudy bag model says

$$|N\rangle_{phys} = Z\,|N\rangle_{bare} + \sqrt{P_{N\pi}}\,|N\pi\rangle + \sqrt{P_{\Delta\pi}}\,|\Delta\pi\rangle$$

with $Z \sim 0.7$, $P_{N\pi} \sim 0.20 - 0.25$ and $P_{\Delta\pi} \sim 0.10 - 0.05$

This pion cloud correction changes the spin observable as follows:

$$\Sigma \to \left\{Z - \frac{1}{3}P_{N\pi} + \frac{5}{3}P_{\Delta\pi}\right\}\,\Sigma\,,$$

i.e. a reduction by a factor of 0.7 to 0.8.

P906 at Fermilab or J-PARC?

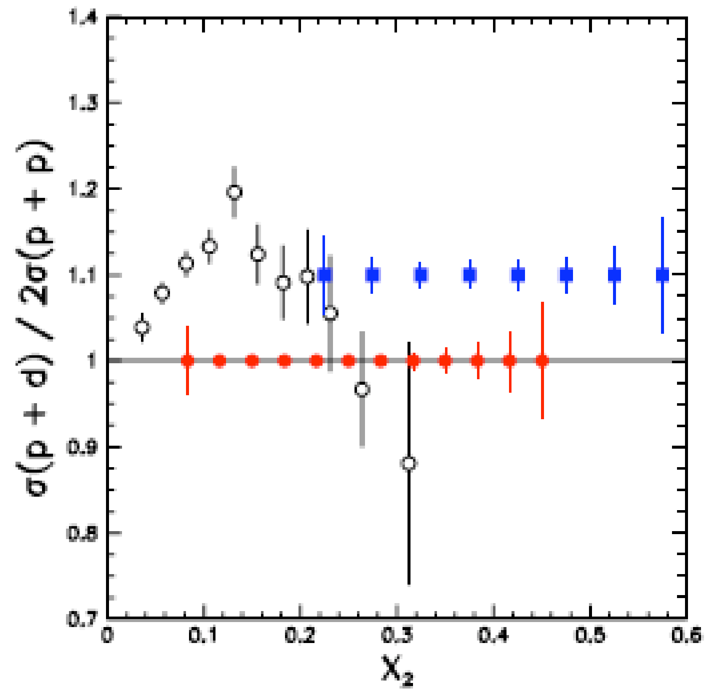


Figure 4: $(p+d)/(p+p)$ Drell-Yan ratios from E866 (open circles) are compared with the expected sensitivites at the 120 GeV Main Injector (solid circles) and the 50-GeV J-PARC (solid squares).

CTEQ6 parton distributions

$$x\,f(x,Q_0) = A_0\,x^{A_1}\,(1-x)^{A_2}\,e^{A_3 x}\,(1+e^{A_4}\,x)^{A_5}$$

	A_0	A_1	A_2	A_3	A_4	A_5
d_v	1.4473	0.6160	4.9670	−0.8408	0.4031	3.0000
u_v	1.7199	0.5526	2.9009	−2.3502	1.6123	1.5917
g	30.4571	0.5100	2.3823	4.3945	2.3550	−3.0000
$\bar{u}+\bar{d}$	0.0616	−0.2990	7.7170	−0.5283	4.7539	0.6137
$s=\bar{s}$	0.0123	−0.2990	7.7170	−0.5283	4.7539	0.6137
$\bar{d}/\bar{u}$	33657.8	4.2767	14.8586	17.0000	8.6408	–

Outline

- statistical model
- other hadrons
- strangeness

Focus on student projects

- The academic context – an undergraduate institution
- Motivation – what aspects of hadron structure are accessible to undergraduates? What models can be used?
- Some work done successfully on MCM - but really appropriate for Ph.D. theses
    - Parton distributions – go beyond CQM
    - Experimental challenge – light sea asymmetry in proton
    - Statistical model – need only NR quantum mechanics
    - Counting partons – zero'th moments
    - Momentum distributions
    - Extensions to other hadrons

Experimental challenge

- From DIS and Drell-Yan: light flavor sea asymmetry in the proton

$$\bar{d} > \bar{u}$$

- Usual explanation - pion cloud model – Thomas, Miller
  - Proton can fluctuate to $\pi^{+}n$, so scattering from dbars in the π^{+} is seen. Fluctuation to $\pi^{0}p$ produces equal numbers of ubars and dbars
  - Many improved meson cloud models

Statistical model – an alternative

- Proposed by Zhang, Zhang and Ma,, Phys. Lett. B 523 (2001) 260. Uses Fock state expansion of the proton in terms of quark and gluon states, together with detailed balance between states.
- Includes quark-gluon splitting and recombination; quark-antiquark creation and annihilation; gluon splitting
- No free parameters
- calculated zero-th moment of light antiquark flavor asymmetry agrees with E866 experiment:

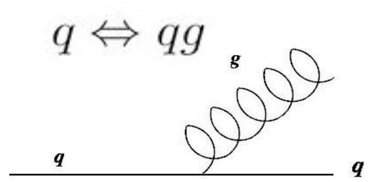
theory: $\bar{d} - \bar{u} = 0.124$ experiment: $\bar{d} - \bar{u} = 0.118 \pm 0.012$

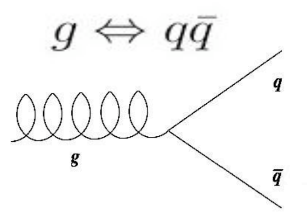
Fock state expansion

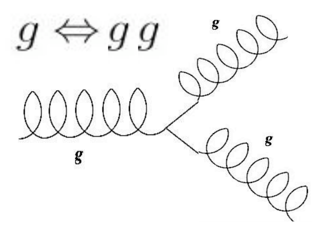
$$|\mathrm{p}\rangle \;=\; \sum c_{i,j,k} \left|\{uud\},\{i,j,k\}\right\rangle >, \qquad \rho_{i,j,k} = \left|c_{i,j,k}\right|^2$$

in which $\{uud\}$ represents the valence quarks and $\{i,j,k\}$ represents the number of u-ubar pairs, d-dbar pairs, and gluons, respectively.

Processes included:







detailed balance

$$\rho_A R_{A\to B} = \rho_B R_{B\to A}$$

in which the rates R are determined by the number of partons that can split or recombine:

$$|uudg\rangle \underset{1\times 3}{\overset{1}{\rightleftharpoons}} |uud\bar{u}u\rangle \qquad |uudg\rangle \underset{1\times 2}{\overset{1}{\rightleftharpoons}} |uud\bar{d}d\rangle \qquad |uud\rangle \underset{1\times 3}{\overset{3}{\rightleftharpoons}} |uudg\rangle$$

The relative probabilities of Fock state components are then determined:

$$\frac{\rho_{ijk}}{\rho_{000}} = \frac{1}{i!(i+2)!j!(j+1)!k!}$$

and an excess of dbar (j) over ubar (i) states in the proton sea results:

$$\bar{d} - \bar{u} = 0.124$$

Fock state probabilities (from Zhang *et al.*)

Table 1

The probabilities, $\rho_{i,j,k}$, of finding the quark–gluon Fock states of the proton, calculated using the principle of detailed balance without any parameter. $|\{q\}, \{i, j, k\}\rangle$ is the subsemble of Fock states, i is the number of $u\bar{u}$ quark pairs, j is the number of $d\bar{d}$ pairs, and k is the number of gluons

i	j	$\|\{q\}, \{i, j, 0\}\rangle$	$\rho_{i,j,0}$	$\rho_{i,j,1}$	$\rho_{i,j,2}$	$\rho_{i,j,3}$	$\rho_{i,j,4}$	$\cdots$
0	0	$\|uud\rangle$	0.167849	0.167849	0.083924	0.027975	0.006994	$\cdots$
1	0	$\|uud\bar{u}u\rangle$	0.055950	0.055950	0.027975	0.009325	0.002331	$\cdots$
0	1	$\|uud\bar{d}d\rangle$	0.083924	0.083924	0.041962	0.013987	0.003497	$\cdots$
1	1	$\|uud\bar{u}u\bar{d}d\rangle$	0.027975	0.027975	0.013987	0.004662	0.001166	$\cdots$
0	2	$\|uud\bar{d}d\bar{d}d\rangle$	0.013987	0.013987	0.006994	0.002331	0.000583	$\cdots$
2	0	$\|uud\bar{u}u\bar{u}u\rangle$	0.006994	0.006994	0.003497	0.001166	0.000291	$\cdots$
1	2	$\|uud\bar{u}u\bar{d}d\bar{d}d\rangle$	0.004662	0.004662	0.002331	0.000777	0.000194	$\cdots$
2	1	$\|uud\bar{u}u\bar{u}u\bar{d}d\rangle$	0.003497	0.003497	0.001748	0.000583	0.000146	$\cdots$
0	3	$\|uud\bar{d}d\bar{d}d\bar{d}d\rangle$	0.001166	0.001166	0.000583	0.000194	0.000049	$\cdots$
3	0	$\|uud\bar{u}u\bar{u}u\bar{u}u\rangle$	0.000466	0.000466	0.000233	0.000078	0.000019	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\ddots$

Student exercises

- Why is a (valence quark + one-gluon) state as probable as the valence quark state (bare proton?)
- Reproduce the table - only a few lines of Mathematica or MatLab code

Momentum distributions

- Monte Carlo code used to determine momentum distribution for each state of *n* partons in rest frame of proton.

- Sum over all Fock states to get $xf(x)$.

- Students reproduced Zhang et al. results. They used RAMBO Monte Carlo code of Kleiss, Stirling and Ellis. Initial calculations carried out for massless partons.

Counting the ways …

- Need to determine the distribution of momenta for n partons such that their momenta add up to zero in the proton rest frame, and their energies add up to the proton rest mass.

- For three partons (leading term of expansion):

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0, \text{ and } E_1 + E_2 + E_3 = 938 \text{ MeV}$$

Monte Carlo Method

Kleiss et al formulate a Monte Carlo method for calculating phase space volume and consequently probability distributions for n massless or massive particles. The method first generates n random massless 4-momenta q_i^μ, then boosts and scales the q_i^μ to p_i^μ with $\sum_i p_i^0 = w$ and $\sum_i \mathbf{p_i} = \mathbf{0}$ by the conformal transformation. If there are massive particles then we transform again to 4-momenta k_i^μ with mass m_i by

$$\mathbf{k_i} = \xi \mathbf{p_i}$$

$$k_i^0 = \sqrt{m_i^2 + (\xi p_i^0)^2}$$

where ξ satisfies $w = \sum_{i=1}^n \sqrt{m_i^2 + (\xi p_i^0)^2}$.

Using this algorithm with enough iterations we can calculate momentum, energy, and Bjorken-x distributions for any particle in a certain state.

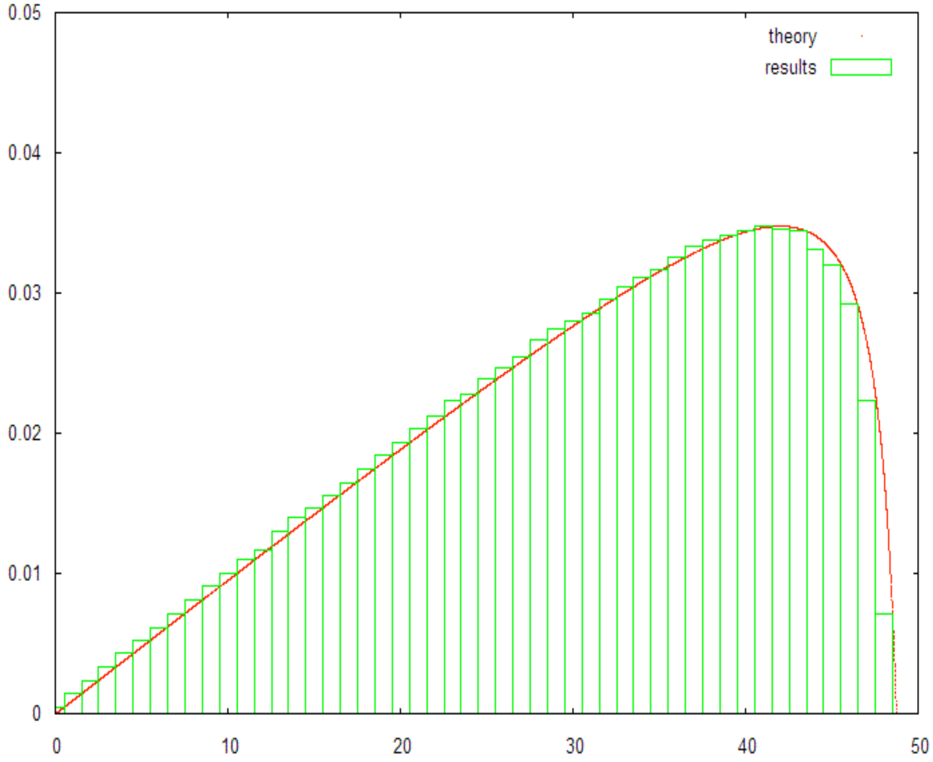
MC distribution compared to theory

Consider a particle with 3 partons, one with mass. The phase space integral is

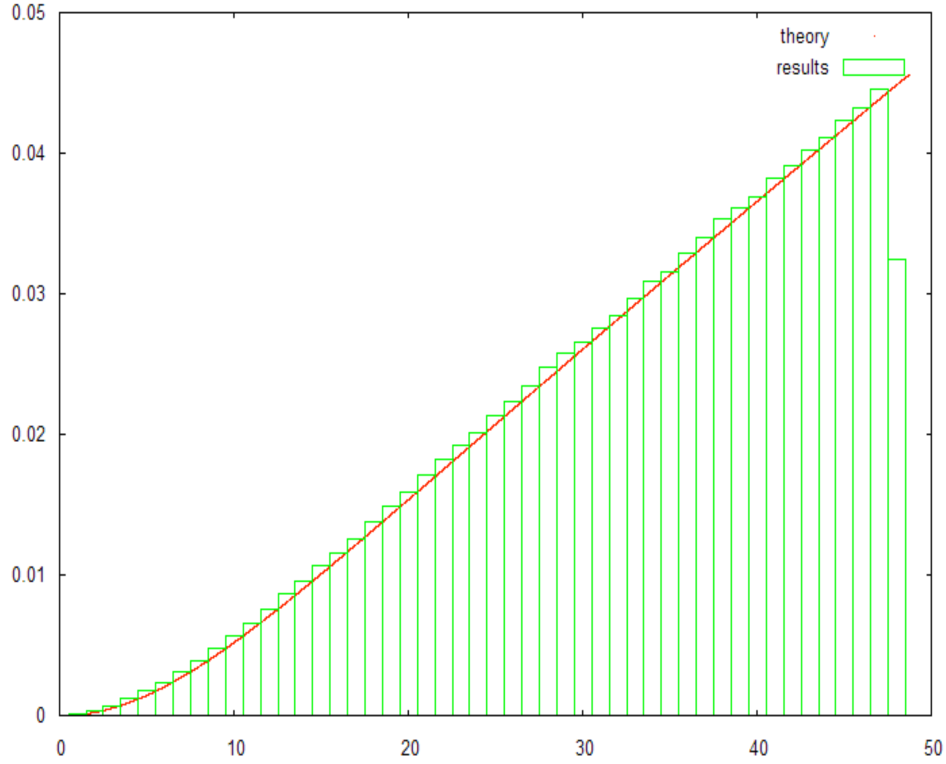
$$V = \int \delta(M - \sum E_i)\delta^3(\sum \mathbf{p_i}) \prod_{i=1}^{3} \frac{d^3 p_i}{2E_i}$$

where $E_1 = \sqrt{p_1^2 + m_1^2}$, $E_2 = p_2$, and $E_3 = p_3$. We calculated this integral both analytically and with the Monte Carlo method and compared the distributions for both p_1 and p_2. The results agreed perfectly.

Momentum distribution for p_1

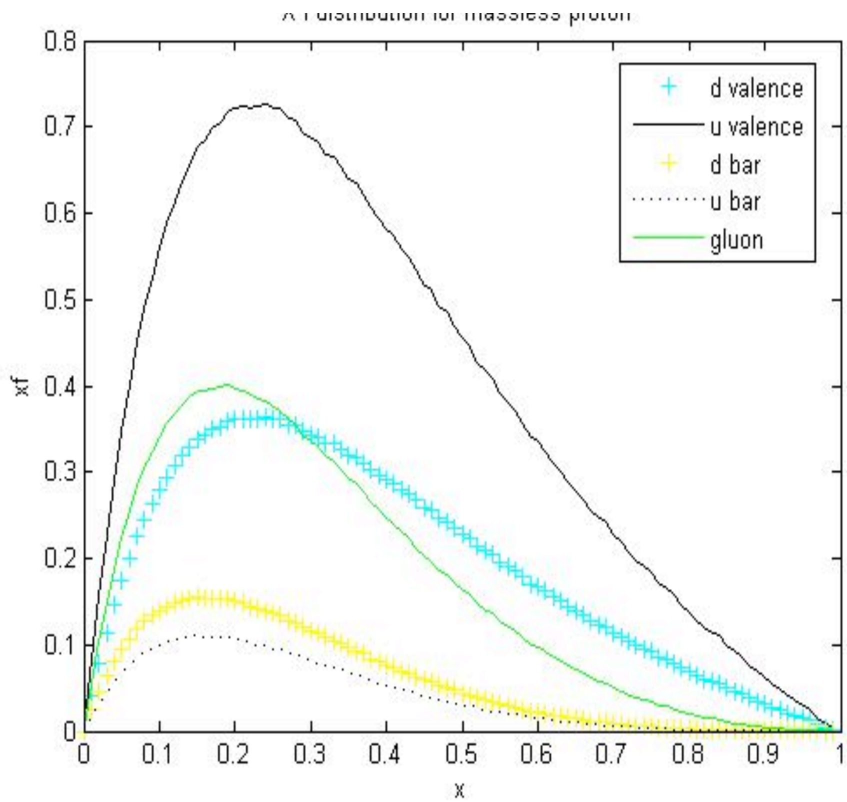


Momentum distribution for p_2



Momentum distributions for the proton

Monte-Carlo calculation of momentum distribution of each *n*-parton state in the proton rest frame; then sum over all Fock states.



Proton sea asymmetry

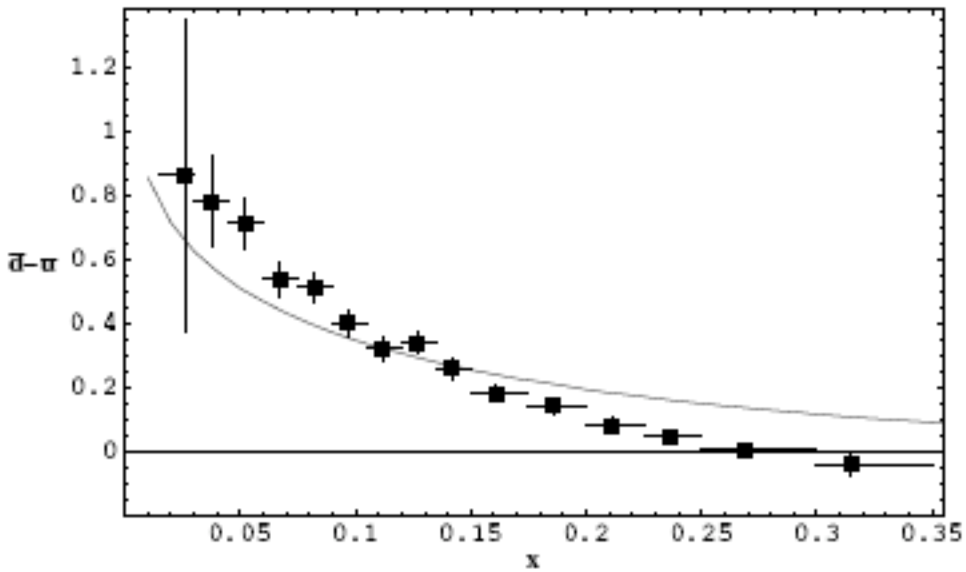


Figure 1: Comparison of statistical model calculation with E866 experimental results [3] for $\bar{d} - \bar{u}$.

ratio

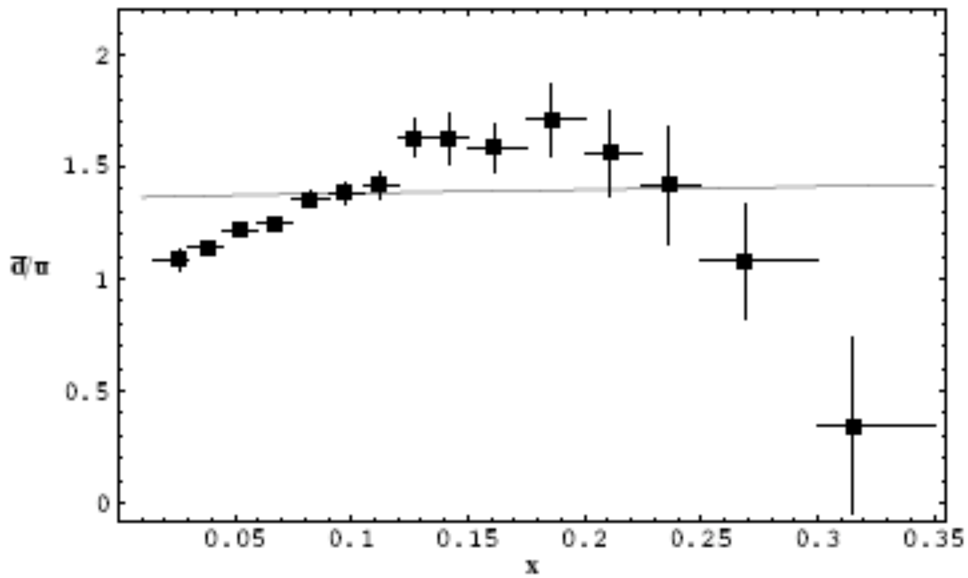


Figure 2: Comparison of statistical model calculation with E866 experimental results [3] for $\bar{d}/\bar{u}$.

- calculated ratio is approximately constant
- need non-perturbative processes (meson cloud)

Extension to the pion

M.A., E.M.H., Phys. Lett. B 611 (2005) 111 - with contributions from Mike Clement

$$|\pi^+ > = \sum_{i,j,k} c_{ijk} |\{u\bar{d}\}\{ijk\} >$$

- leading term in Fock state expansion is 2-parton state

- light quark sea is symmetric

- starting scale determined by requiring that first and second moments of valence quark distribution at $Q^2 = 4$ GeV^2 agree with Sutton et al.

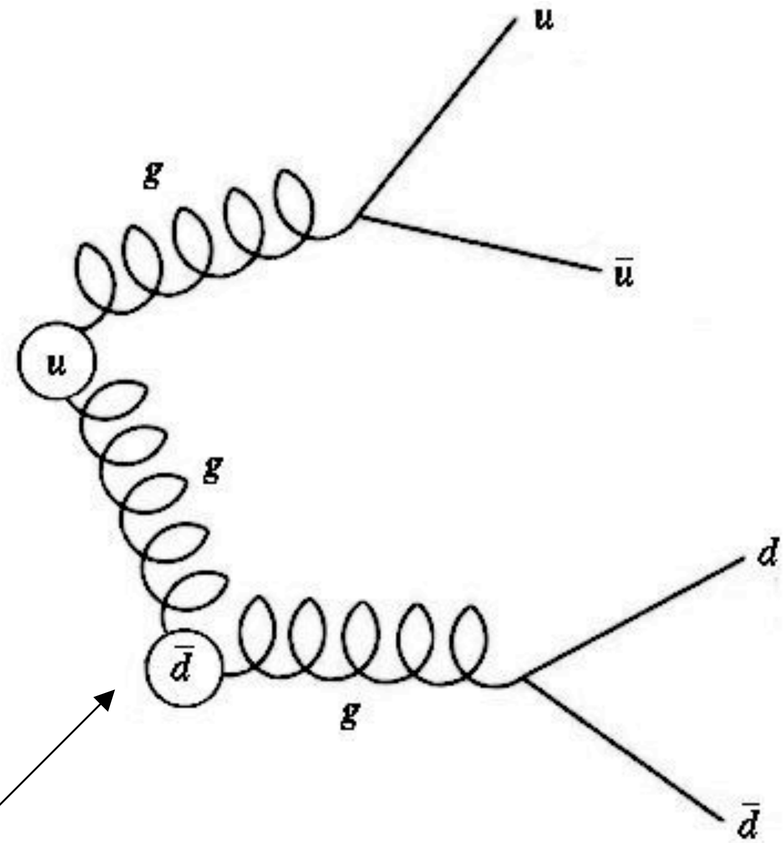
- DGLAP evolution carried out with Kumano's code BF1

Fock state representation

For the Fock state expansion

$$|\pi^+> = \sum_{i,j,k} c_{ijk} |\{u\bar{d}\}\{i,j,k\}>$$

in which i is the number of u-ubar pairs, j the d-dbar pairs, and k the gluons in the sea, we represent the state with 1 u-ubar pair, 1 d-dbar pair, and 3 gluons:



$$|\{u\bar{d}\}\{1,1,3\}> \quad \equiv \quad |u\bar{d}u\bar{u}d\bar{d}ggg>$$

Parton distributions in the pion at Q^2 =1.96 GeV^2

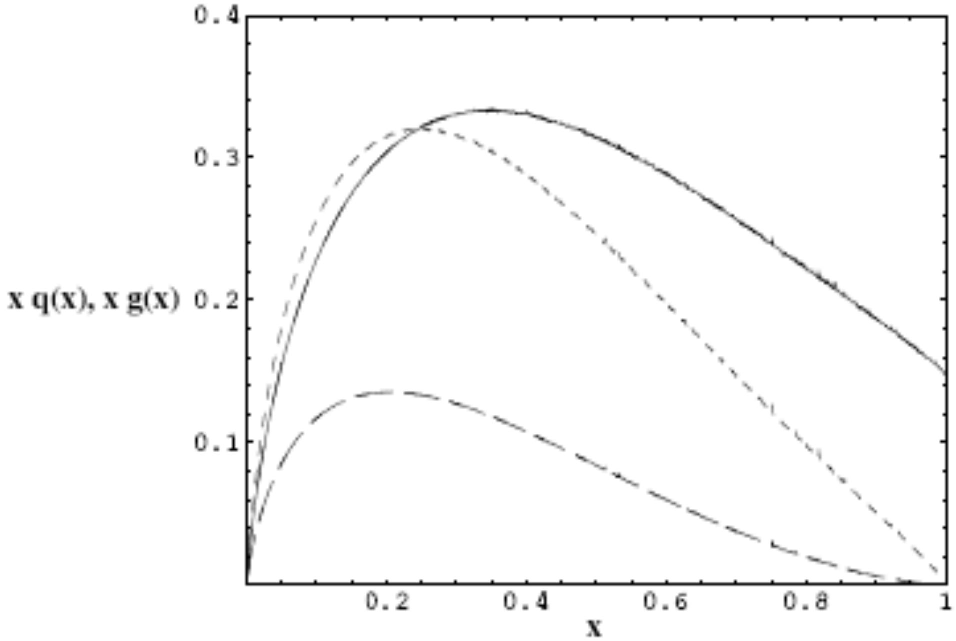


Figure 3: Our results for parton density distributions $x\,q(x)$ and $x\,g(x)$ for the pion. Solid curve: valence quark distribution; long-dashed curve: sea quark distribution; short-dashed curve: gluon distribution.

Pion valence quark distributions

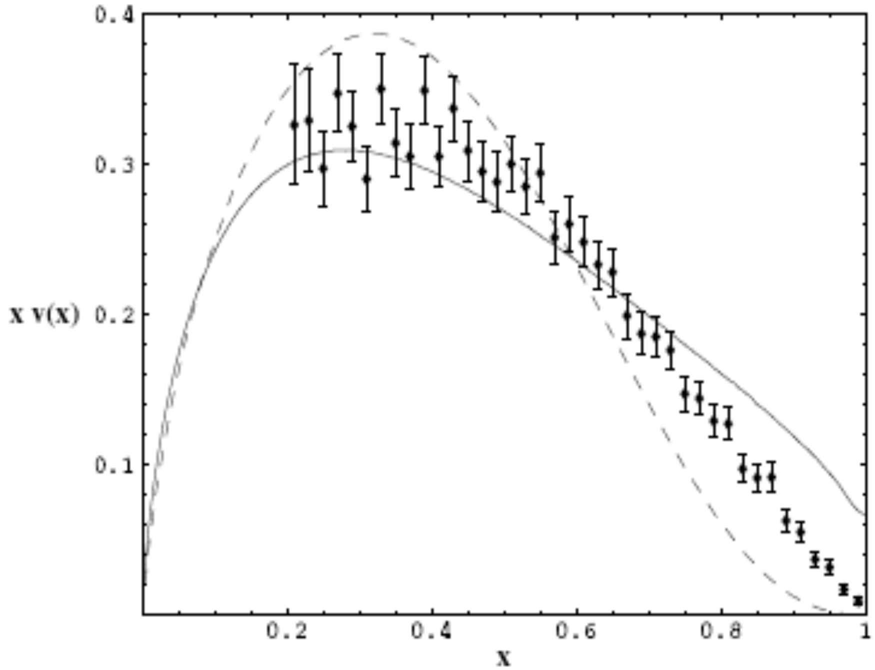


Figure 5: Our results (solid curve) for the valence quark distribution $x\,v(x)$ in the pion, compared to the calculation of Hecht, Roberts and Smith [20] (dashed curve) and the experimental results of Conway et al. [16]. Both calculations were evolved to $Q^2 = 16$ GeV^2 of the E615 experiment.

student projects

- Michael Clement
  - Extension to pion, porting Monte Carlo and evolution codes
  - Poster at DNP, CEU - Tucson, Fall 2003
- Philip Opperman
  - Add strange sea of proton, include mass of strange quarks in Monte Carlo and detailed balance
  - Poster at DNP, CEU - Chicago, Fall 2004
- Sierra Gardner
  - Extension to pentaquark
  - Poster at DNP, CEU - Chicago, Fall 2004
- Blair Garner
  - Extension to kaon
  - Poster at SACNAS, Fall 2005
- Tom Shelly and Stephanie Harp
  - More work on kaon, extension to lambda
  - Posters at DNP, CEU - Nashville, Fall 2006